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Bottom of the Spin-Wave Spectrum in a Magnetic Film

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LINCOLN LABORATORY

BOTTOM OF THE SPIN-WAVE SPECTRUM IN A MAGNETIC FILM

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Group 24

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ABSTRACT

The bottom of the spin-wave spectrum is examined for the case of a thin film magnetized in its plane. The bias field H_b and wave number k_b for the onset of first-order spin-wave instability at high microwave power is calculated. In addition, the critical thickness d_c, at which the uniform mode saturation process changes from first to second order, is obtained. These results are in fairly good agreement with experiment.

Accepted for the Air Force
Stanley J. Wisniewski
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I. INTRODUCTION

Suhl[1] has shown that saturation of the uniform mode in ferromagnetic resonance will occur, at a critical microwave field, via the nonlinear coupling to the uniform mode of pairs of spin waves of energy $\hbar\omega_o/2$, and their ensuing unstable growth. ($\omega_o = 2\pi \times$ drive frequency.) However, if such spin waves do not exist, these "first-order" processes cannot occur, and the uniform mode does not saturate until some higher field at which "second-order" processes appear: growth of spin waves of energy $\hbar\omega_o$. In the latter case at some value of the bias field H, less than its resonance value H_o, spin waves of energy $\hbar\omega_o/2$ again become available and result in a subsidiary absorption. Thus the spin-wave energy at the bottom of the spectrum determines the nature of possible instabilities. As a measure of this bottom, we may take the field H_b, defined as the value of H for which the energy $\hbar\omega_{\vec{k}_b}$ of the lowest-lying spin wave is equal to $\hbar\omega_o/2$.

Calculation of H_b in a bulk ferromagnet is completely straightforward. In a thin film magnetized in its plane, on the other hand, the spin-wave spectrum is badly distorted at low wave numbers, essentially because of the creation of non-cancelling surface poles. It is the purpose of this note to calculate the resultant bottom field and wave number. As a by-product, we obtain the critical thickness at which the transition from first-order to second-order processes takes place. Finally, we compare our theory with experimental results.

II. FIELD AND WAVE NUMBER AT THE BOTTOM OF THE SPECTRUM

The dispersion relation for a film of thickness d = 2L, with a field $\vec{H}$ in the film plane, is given by[2]

$$\omega_{\vec{k}}^2 = [\omega_h + \omega_e k^2 L^2 + \omega_m \chi(kL)][\omega_h + \omega_e k^2 L^2 + \omega_m \tilde{\chi}(kL)\sin^2\Phi] \qquad (1)$$

where

$$\omega_h = H\gamma \qquad (2a)$$

$$\omega_e = \frac{2A}{M_o L^2} \qquad (2b)$$

$$\omega_m = 4\pi M_o \gamma \qquad (2c)$$

$$\chi(\varkappa) = \varkappa^{-1} e^{-\varkappa} \sinh \varkappa \qquad (2d)$$

$$\tilde{\chi}(\varkappa) = 1 - \chi(\varkappa) \qquad (2e)$$

and the wave vector $\vec{k}$ lies in the film plane at an angle Φ to $\vec{H}$. (A = exchange constant, M_o = saturation magnetization, γ = |gyromagnetic ratio|.) This dispersion law is based on a "thin-film approximation," which assumes excitations that are constant across the film thickness. Equation (1) reduces to the bulk spectrum as $kL \to \infty$ ($\chi \to 0$, $\tilde{\chi} \to 1$); in the thin-film limit $kL \to 0$, $\chi \to 1$ and $\tilde{\chi} \to kL$. The term $\omega_m \chi$ results from surface poles, which tend to cancel as $kL \to \infty$. The term $\omega_m \tilde{\chi} \sin^2\Phi$ results from volume poles, whose density vanishes as $kL \to 0$.

The field H_b and wave vector $\vec{k}_b$ at the bottom of the spectrum are given by Eq. (1) and

$$\frac{\partial \omega_{\vec{k}}}{\partial k} = 0 \qquad (3a)$$

$$\Phi = 0 \,, \qquad (3b)$$

which become

$$\omega^2 = [\omega_b + \omega_e \varkappa^2 + \omega_m \chi(\varkappa)][\omega_b + \omega_e \varkappa^2] \qquad (4a)$$

$$2\omega_e \varkappa [\omega_b + \omega_e \varkappa^2 + \omega_m \chi(\varkappa)] + [2\omega_e \varkappa + \omega_m \chi'(\varkappa)][\omega_b + \omega_e \varkappa^2] = 0 \qquad (4b)$$

where

$$\omega = \omega_{\vec{k}_b} \tag{5a}$$

$$\omega_b = H_b \gamma \tag{5b}$$

$$\varkappa = k_b L \quad . \tag{5c}$$

(Note that ω/π is the drive frequency.)

An exact analytical solution to Eqs. (4) for $\varkappa$ and ω_b is impossible to obtain. However, good approximations may be found in the thin-film ($\varkappa << 1$) and thick-film ($\varkappa >> 1$) limits; a numerical solution may be used for intermediate thicknesses.

A. <u>Thin-Film Limit</u>

With the definitions

$$\mu = \frac{\omega_m}{\omega_e} \tag{6a}$$

$$\beta = \frac{\omega}{\omega_m} \tag{6b}$$

$$b = \frac{\omega_b}{\omega_m} \tag{6c}$$

$$\eta = \varkappa \mu^{-1} \tag{6d}$$

Eqs. (4) become

$$b^2 + b\chi - \beta^2 + \mu\eta^2(2b + \chi) + \mu^2\eta^4 = 0 \tag{7a}$$

$$b\chi' + 2\eta(2b + \chi) + \mu\eta^2(4\eta + \chi') = 0 \tag{7b}$$

where [from the $\varkappa << 1$ expansion of Eq. (2d)]

$$\chi = 1 - \mu\eta + \frac{2}{3}\mu^2\eta^2 + \ldots \qquad (8a)$$

$$\chi' = -1 + \frac{4}{3}\mu\eta - \mu^2\eta^2 + \ldots \qquad (8b)$$

We solve Eqs. (7) for b and η by a perturbation expansion in the parameter μ by assuming

$$b = \sum_{n=0}^{\infty} b_n \mu^n \qquad (9a)$$

$$\eta = \sum_{n=0}^{\infty} \eta_n \mu^n \quad . \qquad (9b)$$

Then, with the help of Eqs. (6d) and (8), we equate to zero coefficients of each power of μ in Eqs. (7) to obtain

$$b_o = \frac{1}{2}\left(\sqrt{1 + 4\beta^2} - 1\right) \qquad \eta_o = \frac{1}{4}\left(1 - \frac{1}{\sqrt{1 + 4\beta^2}}\right)$$

$$b_1 = \eta_o^2 \qquad \eta_1 = \frac{2}{3}\eta_o^2 (1 - 18\eta_o + 24\eta_o^2) \qquad (10)$$

$$b_2 = \eta_o\eta_1 \qquad \text{etc.},$$

to any desired order.

In general, the condition for validity of this solution is

$$\mu \ll 1 \qquad (11a)$$

i.e.,

$$L \ll \frac{A}{\sqrt{2\pi M_o^2}} \equiv \lambda_o = 50\ \text{Å}^* \qquad (11b)$$

*For Permalloy ($A = 10^{-6}$ erg/cm; $4\pi M_o = 10^4$ oe).

However, if

$$4\beta^2 << 1 \tag{12a}$$

i.e.,

$$\frac{\omega}{\pi} << \frac{\omega_m}{2\pi} \simeq 30 \text{ kMcps}^* \tag{12b}$$

then from Eqs. (6) and (10) we obtain

$$\omega_b = \frac{\omega^2}{\omega_m}\left[1 - \beta^2 + 2\beta^4 + \ldots + \frac{1}{4}\sigma(1 - 6\beta^2 + \ldots) + \frac{1}{12}\sigma^2 + \ldots\right] \tag{13a}$$

$$\varkappa = \frac{1}{2}\sigma\left[1 - 3\beta^2 + \ldots + \frac{1}{6}\sigma + \ldots\right] \tag{13b}$$

where

$$\sigma = \mu\beta^2 = \frac{\omega^2}{\omega_e\omega_m} . \tag{14}$$

The conditions for validity are now (12b) and

$$\sigma << 2 \tag{15b}$$

i.e.,

$$L << \frac{\gamma}{\omega} 4\sqrt{\pi A} \simeq 500 \text{ Å}^\dagger \tag{15b}$$

The line at the left in Fig. 1 shows H_b, as given by Eq. (13a), as a function of thickness for Permalloy with $\omega/\gamma = 1530$ oe. The corresponding values of $k_b = \varkappa/L$ are plotted in Fig. 2.

B. Thick-Film Limit

For this case, we let

$$\delta = \mu^{-\frac{1}{3}} = \left(\frac{\omega_e}{\omega_m}\right)^{\frac{1}{3}} \tag{16a}$$

$$\zeta = (\varkappa\delta)^{-1} \tag{16b}$$

† For Permalloy at $\omega/\pi = 8.5$ kMcps.

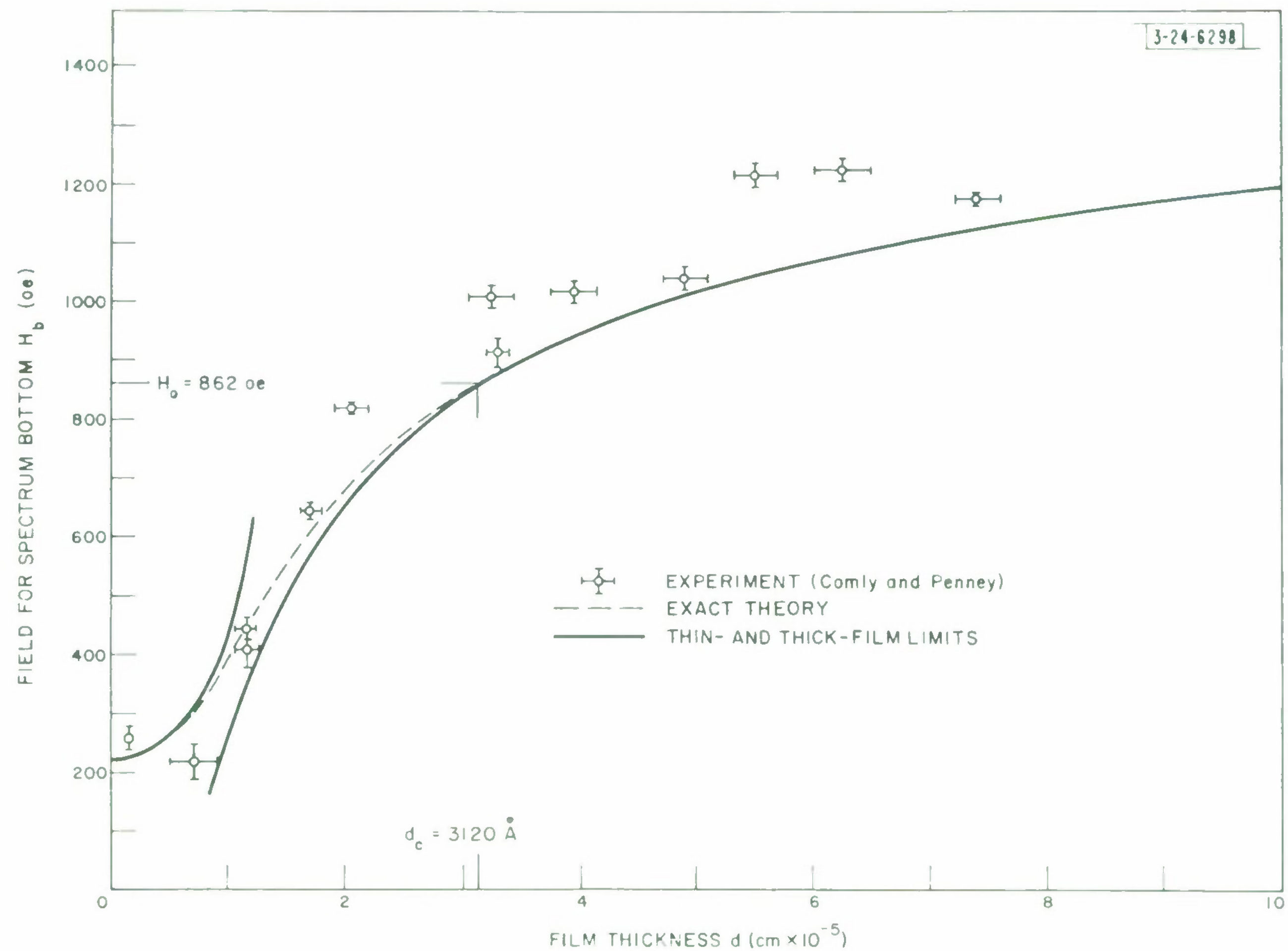

Fig. 1. Field for bottom of the spin-wave spectrum versus film thickness. Theoretical curves are based on the nominal values $A = 10^{-6}$ erg/cm, $4\pi M_o = 10^4$ oe, $\omega/\gamma = 1530$ oe. The experiments were performed on 83% Ni-17% Fe films at about 8.56 kMcps.

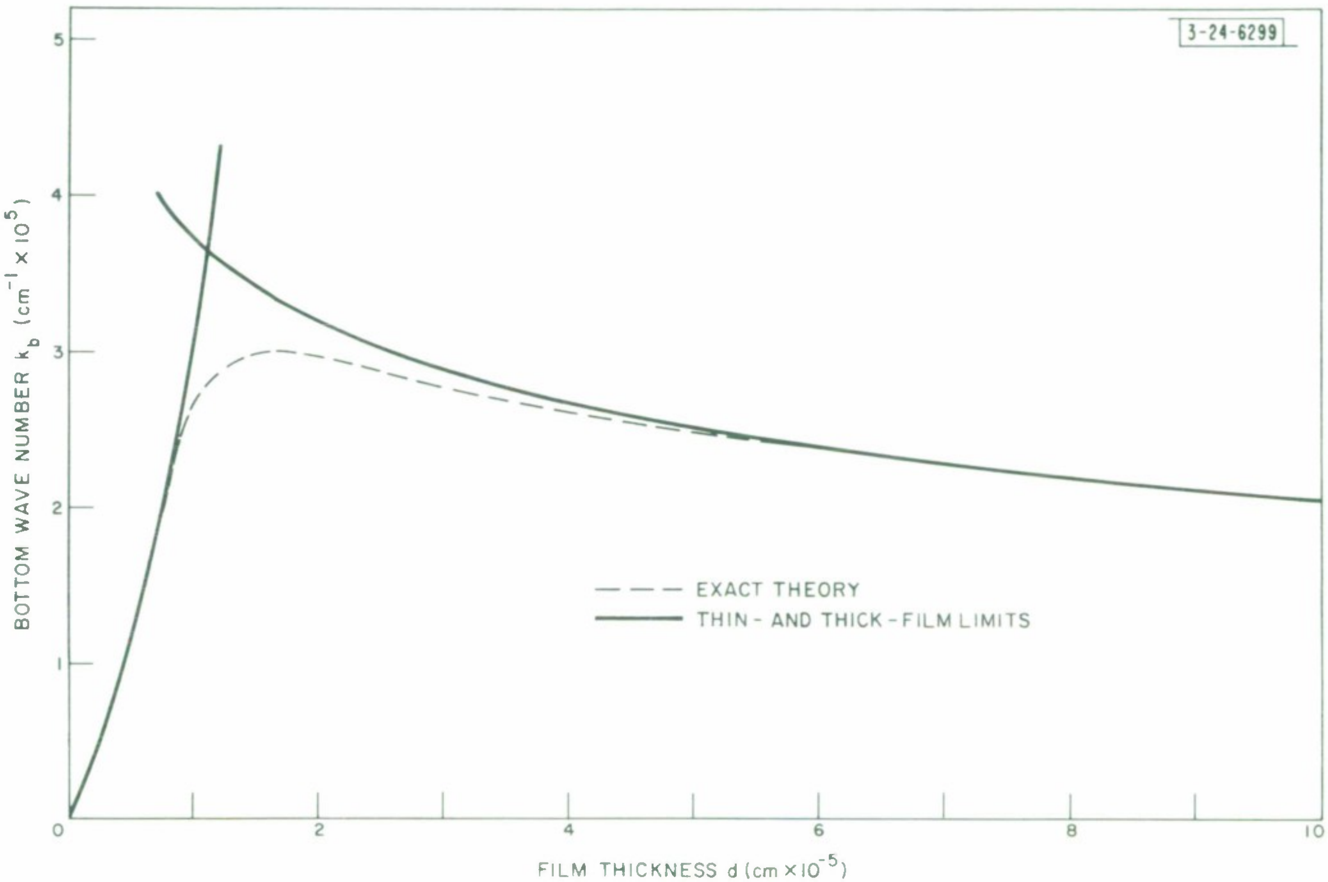

Fig. 2. Wave number of lowest lying spin wave versus film thickness at $H = H_b$. Film parameters are the same as in Fig. 1.

so that Eqs. (4) become

$$\zeta^4(b^2 + b\chi - \beta^2) + \delta\zeta^2(2b + \chi) + \delta^2 = 0 \tag{17a}$$

$$b\chi'\zeta^3 + \delta\chi'\zeta + 2\delta^2\zeta^2 (2b + \chi) + 4\delta^3 = 0 \tag{17b}$$

where [from the $\varkappa >> 1$ expansion of Eq. (2d)]

$$\chi \simeq \frac{1}{2}\delta\zeta \tag{18a}$$

$$\chi' \simeq -\frac{1}{2}\delta^2\zeta^2 \quad . \tag{18b}$$

Proceeding as before, we assume solutions of the form

$$b = \sum_{n=0}^{\infty} b^{(n)}\delta^n \tag{19a}$$

$$\zeta = \sum_{n=0}^{\infty} \zeta^{(n)}\delta^n \tag{19b}$$

and find

$$\begin{aligned} b^{(0)} &= \beta & \qquad \zeta^{(0)} &= 2 \\ b^{(1)} &= -\frac{3}{4} & \qquad \zeta^{(1)} &= \frac{1}{3\beta} \\ b^{(2)} &= \frac{1}{8\beta} & \qquad \zeta^{(2)} &= \frac{1}{6\beta^2} \\ b^{(3)} &= \frac{1}{48\beta^2} & \qquad &\text{etc.} \end{aligned} \tag{20}$$

or

$$\omega_b = \omega(1 - \frac{3}{4}\rho + \frac{1}{8}\rho^2 + \frac{1}{48}\rho^3 + \ldots) \tag{21a}$$

$$\varkappa = \frac{1}{2\delta} (1 - \frac{1}{6}\rho + \frac{1}{12}\rho^2 + \ldots) \tag{21b}$$

where

$$\rho = \frac{\delta}{\beta} = \frac{1}{\omega} (\omega_e\omega_m^2)^{\frac{1}{3}} \quad . \tag{22}$$

The conditions for validity of this solution are

$$\rho << 1 \tag{23a}$$

i.e.,

$$L >> \left(\frac{\gamma}{\omega}\right)^{\frac{3}{2}} 4\pi\sqrt{2M_o A} \simeq 800\ \text{Å}^{\dagger} \tag{23b}$$

and

$$\delta << \frac{1}{2} \tag{24a}$$

i.e.,

$$L >> 2\sqrt{2}\ \lambda_o \simeq 150\ \text{Å}^{*} \tag{24b}$$

Note in the bulk limit $L \rightarrow \infty$ that $\rho \rightarrow 0$ so that $\omega_b \rightarrow \omega$ and $k_b \rightarrow 0$. H_b, as given by three terms of Eq. (21a), is shown by the line on the right in Fig. 1, and k_b as determined from two terms of Eq. (21b) is shown in Fig. 2.

C. Intermediate Case

To find numerical solutions, it is convenient to solve Eq. (4b) for $\omega_b + \omega_e \varkappa^2$ and substitute this into Eq. (4a), to obtain

$$\beta^2 (\mu X' + 4\varkappa)^2 + 2\varkappa X^2 (\mu X' + 2\varkappa) = 0 \tag{25a}$$

$$\omega_b = -\omega_e \varkappa \left(\frac{2\mu X}{\mu X' + 4\varkappa} + \varkappa \right) . \tag{25b}$$

Eq. (25a) may then be solved graphically for $\varkappa$. H_b, as determined from Eq. (25b) is shown by the dashed line in Fig. 1. (Note that the interpolation between the two approximate solutions is smooth.) In Fig. 2 the exact numerical result for the wave number k_b is shown by a dashed line.

III. CRITICAL THICKNESS

The transition from second order to first order instability occurs at a critical thickness $d_c = 2L_c$ such that $\omega_{\vec{k}}$ at the bottom of the spin-wave spectrum is equal to half the uniform precession frequency, or

$$\omega^2 = \left(\frac{\gamma}{2}\right)^2 H_o(H_o + 4\pi M_o) \quad . \tag{26a}$$

$$H_o = H_b(d_c) \quad . \tag{26b}$$

The critical thickness is conveniently found from a plot of H_b vs. d (see Fig. 1). However, an approximate analytic expression for d_c may be obtained from Eqs. (26) and (21a). Using an iteration method, we find

$$d_c = 2\delta_c^{-\frac{3}{2}} \lambda_o \tag{27a}$$

where

$$\delta_c = \frac{4}{3} \frac{\beta(1-3\beta)}{(1+2\beta)} \left\{ 1 + \frac{1}{9} \frac{(1-3\beta)(2+13\beta)}{(1+2\beta)^2} \left[1 + \frac{1}{9} \frac{(1-3\beta)(2+13\beta)}{(1+2\beta)^2} \right]^2 \right.$$

$$\left. + \frac{4}{81} \frac{(1-3\beta)(1-7\beta)}{(1+2\beta)^3} + \ldots \right\} \tag{27b}$$

provided $\beta < 1/3$. If $\beta > 1/3$, first-order processes cannot occur for any thickness, including bulk samples. In Fig. 3 we plot $\delta_c^{3/2}$, as given by three terms of Eq. (26b), vs. β, with scales at top and right for Permalloy. An interesting feature is the existence of an absolute minimum critical thickness given by

$$(d_c)_{min} = 61.6\ \lambda_o = 3080\ \text{Å}^* . \tag{28}$$

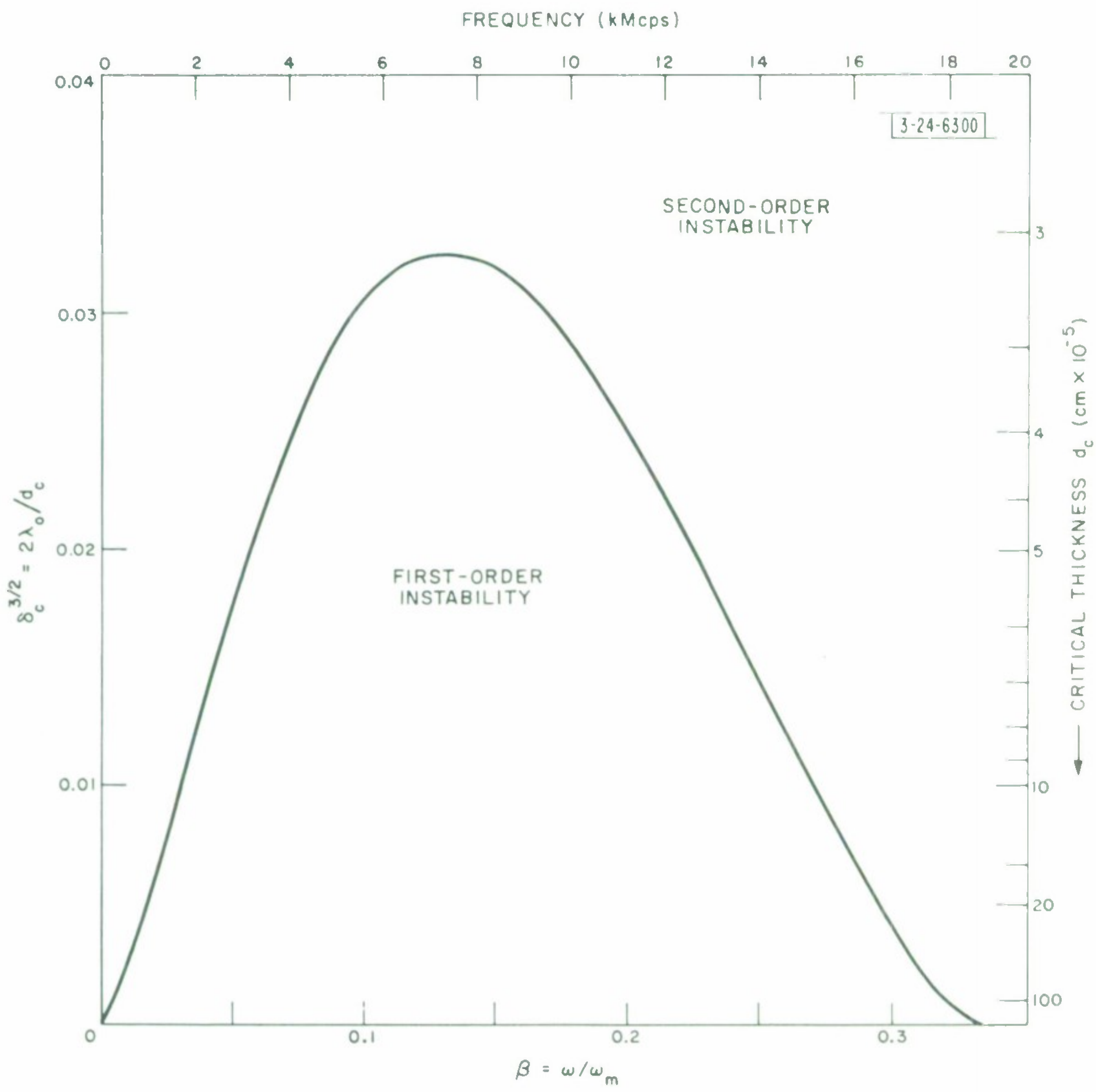

Fig. 3. Normalized inverse critical thickness versus normalized drive frequency. The scales at right and top are computed for $A = 10^{-6}$ erg/cm, $4\pi M_o = 10^4$ oe.

IV. COMPARISON WITH EXPERIMENT

Comly and Penney[3] have measured H_b at 8.56 kMcps in Permalloy films varying in thickness from 150 Å to 7400 Å. Their results are shown in Fig. 1, and are in fairly good accord with theory. The discrepancy probably results from the approximations used to derive Eq. (1): the true normal modes will vary somewhat across the film thickness, lowering the spectrum and therefore raising H_b, particularly for the thicker samples. Comly and Penney also found $2050 < d_c < 3250$ Å; the theoretical value of 3120 Å is consistent with these limits. A more complete account of these experimental results will be published in the near future.[4]

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4. J. B. Comly, T. Penney, K. J. Harte, and R. V. Jones (to be published).

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13. ABSTRACT

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14. KEY WORDS

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